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THE GROWTH OF A CRACK FROM THE BOUNDARY OF A NARROW CAVITY IN A BIAXIALLY-COMPRESSED BODY†

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The problem of the biaxial non-equicomponent compression of a plane with two rectilinear cuts is considered as a model of the growth of a crack from the boundary of a narrow cavity. The problem reduces to a singular integral equation for the displacement jump density at one of the cuts (the crack). An analytic solution of the equation is constructed by reducing the problem to the conjugate problem which arises when the Mellin transform is used and by employing a small parameter which describes the relative length and position of the cuts. Expressions for the stress intensity factors at the vertex of the crack are obtained for the cases of shear and tear-shear rupture. The Griffith–Irwin approach is used to find the angles of orientation of the crack for which its growth is unstable.

1. STATEMENT OF THE PROBLEM

Suppose we have an unbounded linearly elastic body with a narrow rectilinear cavity (slit) under conditions of plane strain (or in a plane stressed state) under biaxial non-equicomponent compression at infinity, such that the surfaces of the cavity are not in contact with one another. It is assumed that along certain directions the material has reduced resistance to shear and stretching. Because the stresses are concentrated near the slit, a crack can grow from its boundary along one of the weakened directions. The growth of the crack may then be unstable, its length growing dynamically without any variation in the external loads.

We will consider an elastic plane containing two rectilinear cuts $L_1\{z = a + lte^{i\alpha}, |a| < L, 0 < t \leqslant 1, 0 < \alpha \leqslant \pi\}$, $L_2\{|\operatorname{Re} z| < L, \operatorname{Im} z = 0\}$, where $z = x + iy$ is a point in the complex plane, l is the length of the cut, L_1 and L is the half-length of the cut L_2. The plane at infinity is acted upon by compressive stresses $\sigma_1 = \lambda\sigma$, $\sigma_2 = \sigma$ ($\sigma < 0$, $\lambda \geqslant 0$) acting at an angle β ($0 \leqslant \beta < \pi/2$) to the x and y coordinate axes respectively (Fig. 1). Across the cuts the normal and shear stresses are continuous (and vanish at the cut L_2), whereas the displacements have jumps.

A system of four real singular integral equations for the unknown displacement jump densities at the cuts can be written out directly using integral representations for the complex potentials in terms of the displacement jumps (as is done, for example, for broken cracks [1]). This uses the Kolosov formula [2] with specified boundary conditions (for the stresses) on the L_1 cut. To reduce the system we use a superposition of two auxiliary problems. Problem A is for a plane with a cut L_2 along which there is an arbitrary displacement jump and specified stresses at infinity. Problem B is for a plane with null conditions at infinity with a cut L_2 on which the normal and shear stresses are specified by putting minus signs in front of the corresponding stresses in problem A.

The complex potentials corresponding to problem A have the form [1]

$$\Phi_A(z) = \sigma\frac{1+\lambda}{4} + \frac{1}{2\pi}\int_{L_2}\frac{g(t)dt}{t-z}$$

$$\Psi_A(z) = \sigma\frac{\lambda-1}{2}e^{-2i\beta} + \frac{1}{2\pi}\int_{L_2}\frac{\overline{g(t)}\,\overline{dt}}{t-z} - \frac{\bar{t}g(t)dt}{(t-z)^2} \tag{1.1}$$

$$g(t) = \frac{2\mu}{1+\kappa}\frac{d}{dt}[\upsilon(t) - iu(t)]e^{i\alpha}$$

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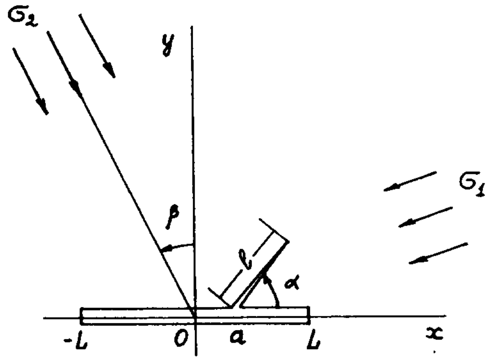

Fig. 1.

Here $g(t)$ is a function proportional to the displacement jump density at the cut L_1, μ is the shear modulus, $\kappa = 3 - 4\nu$ holds for plane deformation and $\kappa = (3 - \nu)/(1 + \nu)$ for the plane stressed state, ν is Poisson's ratio, $\upsilon(t)$, $u(t)$ are the normal and tangential components of the displacement vector, and the square brackets denote the jump of a quantity across the contour.

Taking into account the single-valuedness of the displacements after a circuit of the combined contour $L_1 + L_2$ and using the known solution for an arbitrarily loaded cut [2], the appropriate integrals can be evaluated and the complex potentials in problem B take the form

$$\Phi_B(z) = \frac{1}{4\pi}\int_{L_2}(E_1(z,t)g(t)dt - E_2(z,\bar{t})\overline{g(t)dt}) - \frac{p_0}{2}R(z)$$

$$\Psi_B(z) = -\frac{1}{4\pi}\int_{L_2}\left((E_1(z,t) + E_2(z,t))g(t)dt - (E_1(z,t) - E_2(z,\bar{t}))\overline{g(t)dt}\right) - \tag{1.2}$$

$$- z\Phi'_B(z) + \frac{p_0 - \overline{p_0}}{2}R(z)$$

$$E_1(z,t) = E(z,t) + E(z,\bar{t}),\quad E_2(z,t) = (t - \bar{t})\frac{\partial E(z,t)}{\partial t}$$

$$E(z,t) = \frac{\sqrt{z^2 - L^2} - \sqrt{t^2 - L^2}}{\sqrt{z^2 - L^2}\,(z - t)},\quad R(z) = 1 - \frac{z}{\sqrt{z^2 - L^2}}$$

$$p_0 = \frac{\sigma}{2}(1 + \lambda + (\lambda - 1)e^{2i\beta})$$

The summed potentials from problems A and B satisfy the boundary conditions at both infinity and on the cut L_2 for an arbitrary displacement jump on the cut L_1, the distribution of which must be found from an appropriate equation which follows from the boundary conditions on L_2.

We will consider the case when the length of the crack is small compared with the length of the slit. Putting $z = a + l\xi e^{i\alpha}$, $t = a + l\eta e^{i\alpha}$ $(0 \leqslant \xi, \eta \leqslant 1)$ and expanding (1.2) in a series in the small parameter

$$\varepsilon = 2lL/(L^2 - a^2) \tag{1.3}$$

we find a relation between the normal stresses N and the shear stresses T and the displacement jump along L_1

$$\frac{1}{\pi}\int_0^1\left(M_1\left(\frac{\xi}{\eta}\right)g(\eta) + M_2\left(\frac{\xi}{\eta}\right)\overline{g(\eta)}\right)\frac{d\eta}{\eta} + \varepsilon^2 C + P_0(\xi) = N(\xi) + iT(\xi)$$

$$M_1\left(\frac{\xi}{\eta}\right) = 2e^{i\alpha}\sin\alpha\left(\operatorname{Im} M\left(\frac{\xi}{\eta}\right) - \xi\operatorname{Im} M^2\left(\frac{\xi}{\eta}\right)\right)$$

$$M_2\left(\frac{\xi}{\eta}\right)=\frac{\eta}{\eta-\xi}+\operatorname{Re}M\left(\frac{\xi}{\eta}\right)-2\sin^2\alpha\left(M\left(\frac{\xi}{\eta}\right)-3\xi M^2\left(\frac{\xi}{\eta}\right)+2\xi^2M^3\left(\frac{\xi}{\eta}\right)\right) \tag{1.4}$$

$$M(\xi)=(\xi-e^{-2i\alpha})^{-1},\quad C=\frac{-ie^{i\alpha}\sin^2\alpha}{2\pi}\int_0^1(\sin\alpha\operatorname{Re}g(\eta)+\cos\alpha\operatorname{Im}g(\eta))\eta d\eta$$

$$P_0(\xi)=-\frac{1}{2}\operatorname{Re}p_0R(\xi)+\frac{1}{2}(p_0-\overline{p_0})R(\xi)-ip_0\xi\sin\alpha\frac{\partial}{\partial\xi}R(\xi)$$

The asymptotic behaviour of the stresses near L_2 follows directly from (1.4). Putting $g(\eta)\equiv 0$ we find that

$$\left\|\begin{matrix}\sigma_\alpha(\xi)\\ \tau_{\rho\alpha}(\xi)\end{matrix}\right\|=\frac{\sigma_2\sin\alpha}{\sin\gamma}((1-\lambda)\sin(2\beta-\gamma)I+\varepsilon\xi D)\left\|\begin{matrix}\sin\alpha\\ -\cos\alpha\end{matrix}\right\| \tag{1.5}$$

$$\cos\gamma=\frac{a}{L},\quad D=b(\alpha,\beta)I+b(0,\beta)\left\|\begin{matrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{matrix}\right\|$$

$$b(\alpha,\beta)=\frac{1+\lambda}{2}\sin\alpha+\frac{1-\lambda}{2}\sin(\alpha+2\beta)$$

where I is the unit matrix.

Elementary analysis of relations (1.5) shows that when $(1-\lambda)>0$ $((1-\lambda)<0)$ the L_2: $-L<a<L\cos 2\beta$ part of the upper boundary of L_2 is in a state of extension (compression) and the remaining part $L\cos 2\beta<a<L$ is in a state of compression (extension). The situation is skew-symmetric on the lower edge of the cut.

A crack may develop in the extended zones from the boundary of the cut along a line of weakness, accompanied by separation and shear (a tear-shear rupture), and in the compressed zones a shear crack (a shear rupture) can develop. The appropriate boundary conditions on the cut L_1 take the form

$$N(\xi)+iT(\xi)=0,\quad 0\leqslant\xi\leqslant 1 \tag{1.6}$$

for a tear-shear rupture, and

$$|T(\xi)|=-\operatorname{tg}\rho N(\xi),\quad N(\xi)<0,\quad \operatorname{Re}g=0,\quad 0\leq\xi\leq 1 \tag{1.7}$$

for a shear rupture.

The last condition can only be satisfied for angles α satisfying the inequalities

$$0<\alpha<\pi/2-\rho,\quad \pi/2+\rho<\alpha<\pi \tag{1.8}$$

For hydrostatic compression at infinity there are no extended zones, but the formation of shear cracks according to condition (1.7) is possible at points far from the boundary of the cut with the same restrictions on the angle α.

In the case when $\beta=0$ (the cut is directed along the direction of σ_1) the whole boundary is either under extension (when $|\sigma_2|>|\sigma_1|$) or compression (when $|\sigma_2|<|\sigma_1|$), and conditions (1.6) or (1.7) for crack growth are preserved.

A maximum of the stretching stresses is reached when $\alpha=\pi/2$, and the maximum of $|T|+\operatorname{tg}\rho N$ when $\alpha=\pi/4-\rho/2$, $\alpha=3\pi/4+\rho/2$, so that when there are no lines of weakening cracks are most likely to develop along these directions.

2. THE PROBLEM OF MATCHING

The application of a Mellin transform to integral relation (1.4) leads to a functional relation between the transforms of the stresses and displacement jumps along the line of the cut L_1 which occurs in the strip $\delta<\operatorname{Re}s\leqslant 0$ $(\delta<0)$ of the complex transformation parameter s

$$-\operatorname{tg}\frac{s\pi}{2}G(s)V^-(s)=V^+(s)+F^-(s) \tag{2.1}$$

$$G(s)=\frac{\lambda_1(s)+\lambda_2(s)}{2}I-\frac{\lambda_1(s)-\lambda_2(s)}{2}B \tag{2.2}$$

$$\lambda_{1,2}(s)=(h\pm s\sin\alpha\sin s(\pi-2\alpha)\sqrt{f})\sin^{-2}(s\pi/2)$$

$$h=\sin s\alpha\sin s(\pi-\alpha)-s^2\sin^2\alpha\cos s(\pi-2\alpha),\ \ f=-\det B=1-s^2\sin^2\alpha$$

$$B=\frac{1}{\sqrt{f}}\begin{Vmatrix}\cos\alpha & (1-s)\sin\alpha\\ (1+s)\sin\alpha & -\cos\alpha\end{Vmatrix}$$

$$V^-(s)=\int_0^1\begin{Vmatrix}\operatorname{Re}g(x)\\ -\operatorname{Im}g(x)\end{Vmatrix}x^s dx,\ \ V^+(s)=\int_1^\infty\begin{Vmatrix}\sigma_\alpha(x)\\ \tau_{\rho\alpha}(x)\end{Vmatrix}x^s dx$$

$$F^-(s)=-\frac{\sigma\sin\alpha}{\sin\gamma}\left(\frac{1-\lambda}{1+s}\sin(2\beta-\gamma)I+\frac{\varepsilon}{2+s}D\right)\begin{Vmatrix}\sin\alpha\\ -\cos\alpha\end{Vmatrix}-$$

$$-\frac{\varepsilon^2\sin^2\alpha}{1+s}\begin{Vmatrix}\sin\alpha\\ -\cos\alpha\end{Vmatrix}\|\sin\alpha,\cos\alpha\|V^-(1).$$

Here G is a (2×2) matrix, $\lambda_{1,2}(s)$ are its eigenvalues, and $V^\pm$ and F^- are (2×1) matrices.

In the case of a tear-shear rupture the side of the cut L_1 is free from stresses (boundary condition (1.6)) and relation (2.1) is a problem of matching for two pairs of functions $V^\pm(s)$ analytic in the domains $\operatorname{Re} s\leqslant 0$, $\operatorname{Re} s\geqslant 0$. In the case of shear rupture, when boundary conditions (1.7) are satisfied on L_1, multiplication of (2.1) on the left by the row $(\operatorname{tg}\rho,-1)$, using $\operatorname{Re} g\equiv 0$, gives a one-dimensional matching problem which preserves the form of (2.1), but its coefficient $G(s)$, the unknown functions $V^\pm(s)$, and the free term $F^-(s)$ take the form

$$G(s)=(h+s\sin\alpha\sin s(\pi-2\alpha)(\cos\alpha-(1-s)\operatorname{tg}\rho\sin\alpha))\sin^{-2}(s\pi/2)$$

$$V^-(s)=-\int_1^\infty\operatorname{Im}g(x)x^s dx,\ \ V^+(s)=\int_0^1(\tau_{\rho\alpha}(x)-\operatorname{tg}\rho\,\sigma_\alpha(x))x^s dx \tag{2.3}$$

$$F^-(s)=\frac{-\sigma\sin\alpha\cos(\alpha-\rho)}{\sin\gamma\cos\rho}\left(\frac{1-\lambda}{1+s}\sin(2\beta-\gamma)+\right.$$

$$\left.+\frac{\varepsilon}{2(2+s)}((1+\lambda)\sin\alpha+(1-\lambda)(\sin(\alpha+2\beta)+\frac{\sin 2\beta\cos(2\alpha-\rho)}{\cos(\alpha-\beta)}\right)-$$

$$-\frac{\varepsilon^2\sin^2\alpha\cos(\alpha-\rho)}{2\pi(1+s)\cos\rho}\cos\alpha V^-(1)$$

When $\varepsilon=0$, the boundary-value problems (2.1), (2.2) and (2.1), (2.3) are identical with those considered in [3] and [4], respectively. After factorizing the coefficients $\operatorname{ctg}(s\pi/2)=2K^+(s)K^-(s)/s$, $G(s)=G^+(s)((G^-(s))^{-1}$ the solution can in both cases be represented in the form (choosing the matching line to be the imaginary axis)

$$V^-(s)=-\frac{2K^-(s)}{s}G^-(s)(H-\phi^-(s)),$$

$$V^+(s)=\frac{1}{K^+(s)}(G^+(s))^{-1}(H-\phi^+(s))$$

$$K^\pm(s)=\Gamma(1\mp s/2)/\Gamma(1/2\mp s/2) \tag{2.4}$$

$$\phi^\pm(s)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{\phi(t)}{t-s}dt,\ \ \phi(t)=K^+(t)(G^+(t))^{-1}F^-(t)$$

$$H = Q\begin{Vmatrix} k_1 \\ k_2 \end{Vmatrix}\frac{\sqrt{\pi}}{2}, \quad Q - \lim_{s\to\infty} G^+(s) = \lim_{s\to\infty}(G^-(s))^{-1}$$

where k_1 and k_2 are the stress intensity factors at the vertex of the cut L_1.

Since $V^-(0)$ is the opening of the crack at the point where it reaches the boundary of the cavity, then the (physical) requirement that this quantity should be bounded ($C = \phi^-(0)$) leads to an expression for the stress intensity factors

$$\begin{Vmatrix} k_1 \\ k_2 \end{Vmatrix}\frac{2}{\sqrt{\pi}} \; Q^{-1}\phi^-(0) \tag{2.5}$$

in terms of the boundary value of the function $\phi^-(0)$, which can be calculated from the theorem of residues

$$\phi^-(0) = -\operatorname{res}\phi(-1) - \tfrac{1}{2}\operatorname{res}\phi(-2), \quad \operatorname{res}\phi(-k) = \lim_{t\to -k}(t+k)\phi(t), \quad k = 1,2$$

The unknown constant $V^-(1)$ is determined from the solution, for which it is necessary to put $s = 1$, which leads to the linear equation

$$V^-(1) = 2sK^-(1)G^-(1)(\phi^-(1) - \phi^-(0)) \tag{2.6}$$

In problem (2.1), (2.2) a factorization of the matrix $G(s)$ is sought in the form [3]

$$G^{\pm}(s) = \Delta^{\pm}\left(\operatorname{ch}(\sqrt{f}\gamma^{\pm})I - \operatorname{sh}(\sqrt{f}\gamma^{\pm})B\right) \tag{2.7}$$

$$\Delta^{\pm}(s) = \exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{\ln\sqrt{\Delta(t)}}{t-s}dt\right], \quad \gamma^{\pm}(s) = \exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{\gamma(t)}{\sqrt{f(t)}(t-s)}dt\right]$$

where $\Delta(s) = \lambda_1(s)\lambda_2(s)$, $\gamma(s) = \tfrac{1}{2}\ln(\lambda_1(s)/\lambda_2(s))$ are, respectively, the determinant and index of the matrix $G(s)$.

Take into account the evenness of the real functions $\Delta(it)$, $\gamma(it)$ along the matching line, we can write the functions $\Delta^{\pm}(s)$ $\gamma^{\pm}(s)$ on the real axis in the form

$$\Delta^+(-k) = (\Delta^-(k))^{-1} = \exp\left[\frac{k}{\pi}\int_0^{\infty}\frac{\ln\sqrt{\Delta(it)}}{t^2-k^2}dt\right] \tag{2.8}$$

$$\gamma^+(-k) = -\gamma^-(-k) = \exp\left[\frac{k}{\pi}\int_0^{\infty}\frac{\gamma(it)}{\sqrt{f(it)}(t^2+k^2)}dt\right]$$

The coefficient in the matching problem (2.1), (2.3) has no zeros or poles on the imaginary axis and tends to unity at infinity, which enables us to factorize it in the form [4]

$$G^{\pm}(s) = \exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{\ln G(t)}{t-s}dt\right]$$

Separating the real and imaginary parts of the integrand along the matching line, using the evenness of the modulus ($\arg G$) and the oddness of the argument ($G^+(s)$), we find that on the negative real axis the value of $|G|$ is

$$G^+(-k) = \exp\left[\frac{1}{\pi}\int_0^{\infty}\frac{k\ln|G(it)| + t\arg G(it)}{k^2+t^2}dt\right] \tag{2.9}$$

3. UNSTABLE CRACK GROWTH

During the growth of the crack the sum of the squares of the stress intensity factors is proportional to the density of the energy released by the fracture of the material (see for example [5]), and so we can take

$$|k_1 + ik_2| = k_c \tag{3.1}$$

to be a condition governing the crack growth, where k_c is a constant of the material which describes its crack resistance. From the solution obtained above we have the asymptotic representation for the stress intensity factors

$$k_j = k_{0j} + \varepsilon k_{1j} + \varepsilon^2 k_{2j} + \ldots, \quad j = 1, 2 \tag{3.2}$$

where the functions k_{mj} do not depend on ε.

For small crack lengths the onset of crack growth is governed by the magnitudes of the first terms of the asymptotic form in both the stable and unstable (dynamic) cases. Below we shall assume (starting with the static solution) that instability occurs when the left-hand side of (3.1) increases as the crack length increases, and that in the opposite case the crack grows stably. Ignoring terms in (3.2) of order higher than unity, we can represent the condition for unstable growth in the form

$$\left\| k_{01}, k_{02} \right\| \left\| \begin{matrix} k_{11} \\ k_{12} \end{matrix} \right\| > 0. \tag{3.3}$$

Using formulae (2.2), (2.4)–(2.8) the stress intensity factors in the case of a tear-shear fracture take the form

$$\left\| \begin{matrix} k_1 \\ k_2 \end{matrix} \right\| = \frac{\sigma \sin\alpha}{\sin\gamma} \left((1-\lambda)\sin(2\beta-\gamma) G_1 + \frac{\varepsilon}{\pi} G_2 D \right) \left\| \begin{matrix} \sin\alpha \\ -\cos\alpha \end{matrix} \right\| \tag{3.4}$$

when $(1-\lambda)\sin(2\beta-\gamma) < 0$. Here $G_n = Q^{-1}(G^+(-n))^{-1}$ are matrices corresponding to the dimensionless stress intensity factors $(k_j/\sqrt{l})$ in the case when a constant load ($n = 1$) and a linear load ($n = 2$) are specified on the crack. The elements of the matrices G_n are given in Table 1 as functions of the angle α.

If $\alpha > \pi/2$, the elements of G_n are obtained by replacing α by $\pi - \alpha$ and multiplying by $(-1)^{k+l}$.

Condition (3.3) is not satisfied when $\beta = 0$ and $\beta = \pi/2$ for all the values of λ and α in the range of variation under consideration ($\lambda \geqslant 0$, $0 < \alpha < \pi$). Calculations show that for angles α close to $\pi/2$, relation (3.3) is also not satisfied, i.e. only stable crack growth is possible perpendicular to the boundary of the cavity.

Figure 2 shows regions of values of the parameters α, β and ρ for which the growth of tear-shear cracks is unstable. When $\lambda > 1$ ($\lambda < 1$) inequality (3.3) is satisfied for all points with coordinates α, β lying above (below) the curve with the given value of λ. Curves corresponding to values of λ and λ^{-1} are skew-symmetric about the axes $\alpha = \pi/2$, $\beta = \pi/4$.

For shear fracture we have, from (2.3)–(2.6) and (2.9) that

Table 1

α	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$	$5\pi/12$	$\pi/2$
$\{G_1\}_{1,1}$	5.237	2.339	1.593	1.292	1.160	1.122
$\{G_1\}_{1,2}$	0.477	0.282	0.184	0.113	0.054	0
$\{G_1\}_{2,1}$	3.105	1.041	0.507	0.266	0.117	0
$\{G_1\}_{2,2}$	1.737	1.377	1.236	1.166	1.132	1.122
$\{G_2\}_{1,1}$	4.369	2.278	1.724	1.496	1.395	1.365
$\{G_2\}_{1,2}$	0.440	0.252	0.161	0.098	0.047	0
$\{G_2\}_{2,1}$	2.064	0.694	0.340	0.179	0.079	0
$\{G_2\}_{2,2}$	1.923	1.591	1.465	1.404	1.375	1.365

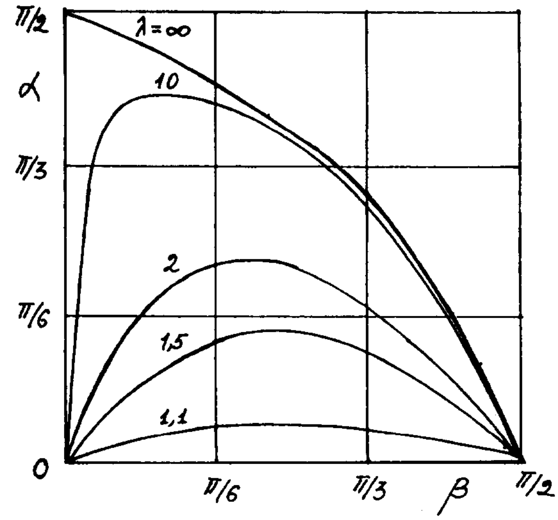

Fig. 2.

$$k_2 = \frac{-\sigma \sin\alpha\cos(\alpha+\rho)}{\sin\gamma\cos\rho}\Big((1-\lambda)\sin(2\beta-\gamma)G^+(-1)+$$

$$+\frac{\varepsilon}{\pi}G^+(-2)\left((1+\lambda)\sin\alpha+(1-\lambda)(\sin(\alpha+2\beta)+\frac{\sin 2\beta\cos(\rho+2\alpha)}{\cos(\alpha+\rho)}\right)\Big) \tag{3.5}$$

when $0 < \alpha \leqslant \pi/2 - \rho$, $(1-\lambda)\sin(2\beta-\gamma) > 0$.

When $\pi/2 + \rho \leqslant \alpha < \pi$ we must change the sign in expression (3.5) and put $\rho = -\rho$.

The dependence of the functions $G^{+-1}(-1)$ and $G^{+-2}(-2)$ on α and ρ is given in Table 2.

The growth of a shear crack is unstable, by condition (3.3), if both terms in (3.5) have the same sign. Figure 3 shows the regions of values of the parameters ρ and α for two values of λ and β for which crack growth is impossible (1) (because of condition (1.8)), stable (2) and unstable (3). Calculations show that for other values of λ, $\beta(\beta \neq 0)$ the situation remains qualitatively similar.

When $\beta = 0$, $\lambda > 1$ the growth of shear crack is unstable for all angles α satisfying condition (1.8). The growth of shear cracks from the end of a slit which models a mine working has been considered [6], and in the case when $\beta = 0$ a range of angles between the crack and the slit has been found for which the crack grows unstably.

It is interesting to note that directions at the end of the working along which shear cracks could not develop unstably could, as the working was extended, become directions along which the cracks should grow unstably, which rather extends the "dangerous" range of angles between geological fractures and workings that was given in [6].

Table 2

	$(G^+(-1))^{-1}$				$(G^+(-1))^{-2}$			
α	$\rho=0$	$\pi/12$	$\pi/6$	$\pi/3$	0	$\pi/12$	$\pi/6$	$\pi/3$
$\pi/12$	1.221	1.196	1.166	1.048	1.117	1.100	1.079	0.992
$\pi/6$	1.132	1.134	1.136	1.135	1.068	1.072	1.076	1.085
$\pi/4$	1.121	1.139	1.161		1.068	1.082	1.099	
$\pi/3$	1.121	1.142	1.170		1.072	1.087	1.106	
$5\pi/12$	1.121	1.135			1.073	1.082		
$\pi/2$	1.121				1.073			

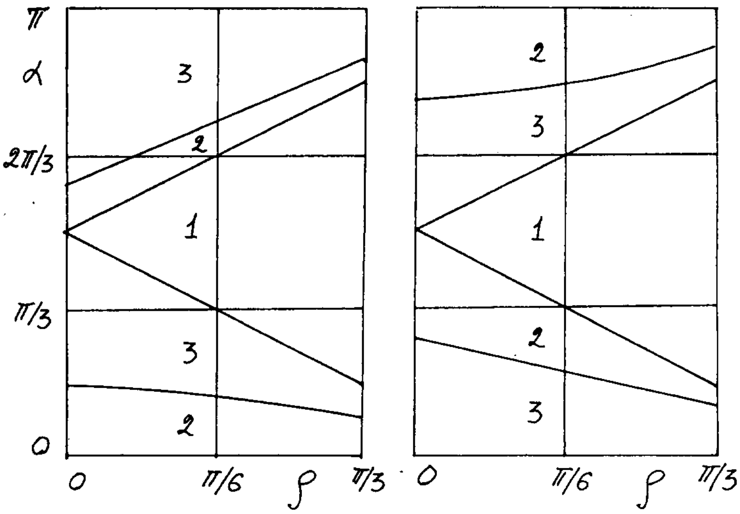

Fig. 3.

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